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Correction to Jin-Ting Zhang's "Approximate and Asymptotic Distributions of Chi-Squared-Type Mixtures With Applications"

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ABSTRACT
Zhang derives approximations for the distribution of a mixture of chi-squared distributions. The two derived approximation bounds in Theorem 1.1 both contain an arithmetic error. These are corrected here.

KEYWORDS
Chi-squared approximation; Chi-squared-type mixtures; Corrections

1. Introduction

Zhang (2005) studied approximate distributions of the chi-squared-type mixture

$$T = \sum_{r=1}^{q} c_r A_r, \qquad A_r \sim \chi_1^2\left(u_r^2\right), \quad \text{independent}, \tag{1}$$

where $c_r, r = 1, 2, \ldots, q$ are nonzero real coefficients and $u_r^2, r = 1, 2, \ldots, q$ are the noncentral parameters of the chi-squared variates $A_r, r = 1, 2, \ldots, q$. The normalized version T^* of this mixture T is approximated, both by a standard normal distribution Z, and by a single chi-squared distribution R^* (Zhang, 2005, Eq. (6)). Denoting the density function of a random variable X by $f_X(x)$, and denoting the standard normal case $f_Z(x)$ by $\phi(x)$, Zhang (2005) derived the following approximation bounds in his Theorem 1:

Theorem 1.1.

(a) For the normal approximation of T, when $\Delta < 1/8$,

$$\sup_x \left| f_{T^*}(x) - \phi(x) \right| < 0.1323 \left\{ 4 + \frac{.2803}{(1 - 8\Delta)^2} \right\} d^{*-1/2}. \tag{2}$$

(b) For the chi-squared approximation of T, when $\Delta < 1/10$,

$$\sup_x \left| f_{T^*}(x) - f_{R^*}(x) \right| < 0.1403 \left[\left\{ 3 + \frac{0.1572}{(1 - 10\Delta)^{5/2}} \right\} M + \left\{ 3 + \frac{0.1572}{(1 - 10/d)^{5/2}} \right\} / d \right]. \tag{3}$$

Here, Equation (2) corresponds to (Zhang, 2005, Equation (12)) and Equation (3) corresponds to (Zhang, 2005, Equation (13)). These equations contain variables that are undefined in this particular correction; their definitions are irrelevant to the discussion at hand.

2. Arithmetic Errors

The floating point constants that are present in Theorem 1.1 are derived in the Proof of Theorem 1.1, in Zhang (2005, Appendix B). Both the factors directly following the $<$ sign are correct, but the factors in the numerator of the fractions are erroneously derived. P. 283 of Zhang (2005) features the line "The assertion (a) follows by noting that [...] $8!^{1/4}/6 = .2803$." In fact, $2.3617 < 8!^{1/4}/6 < 2.3618$. P. 284 of Zhang (2005) features the line "The assertion (b) follows by noting that [...] $10!^{1/4}/(2^{7/2}) = .1572$." In fact, $3.8577 < 10!^{1/4}/(2^{7/2}) < 3.8578$.

3. Correction

Rounding the numbers up to the fourth decimal, the first half of Zhang (2005, Theorem 1) should read:

Theorem 3.0.

(a) For the normal approximation of T, when $\Delta < 1/8$,

$$\sup_x \left| f_{T^*}(x) - \phi(x) \right| < .1323 \left\{ 4 + \frac{2.3618}{(1 - 8\Delta)^2} \right\} d^{*-1/2}.$$

(b) For the chi-squared approximation of T, when $\Delta < 1/10$,

$$\sup_x \left| f_{T^*}(x) - f_{R^*}(x) \right| < .1403 \left[\left\{ 3 + \frac{3.8578}{(1 - 10\Delta)^{5/2}} \right\} M + \left\{ 3 + \frac{3.8578}{(1 - 10/d)^{5/2}} \right\} / d \right].$$

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Reference

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