

# Immobile Random Sensor Networks for Surveillance and Rescue

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## Abstract

Networks consisting of millions of cheap sensors can be used to watch over areas for intruders, fires and similar threats. In the work presented here, we have used computer simulations to study some possibilities of using random networks. The relative position of the sensors can only be determined by the sensors through contact with nearby sensors. In this study, we performed simulations on very large networks with a variety of homogeneous and inhomogeneous sensor distributions, to determine whether the number of communication steps between non-neighboring sensors can be used to estimate geometric distance between them. Our results indicate that this requires a relatively large sensor density. Also, inhomogeneities in communication range provide a problem. But sensor density need not be constant across the network, as long as the minimum (local) density is sufficient. After correction for the average density, accurate distance estimates were obtained.

## 1 Introduction

Wireless networks with a large number of tiny intelligent sensors are useful for applications that need to monitor large areas, in particular when it is not possible to reach those areas or when it is too dangerous to do so. It is often not possible to arrange the sensors in regular grids, for instance if one wants to place many sensors in a large forest, for a fire warning system. One might decide to drop sensors from the air. Depending on weather conditions the density may be more or less constant and a rough estimate of the sensor density will be possible. When the sensors are spread over large areas and have to perform their surveillance task for an extended period of time, they should consume as little power as possible, even if the sensors are equipped with solar cells. Communication between sensors should be at low power and infrequent, and therefore sensors will be able to reach only nearby sensors. In military applications, this may also be necessary to prevent detection of the network. Depending on the application, the average distance between neighboring sensors may vary from one meter to several hundred meters.

Usually the problem with randomly distributed sensors is the localizations of the sensors [1-5]. In this paper, we assume that the sensors have sufficient memory and computing power, to be able to do extensive calculations and to store sophisticated ontologies. With the current state of the art technology, this is quite feasible for sensors of only a few centimeters in size. In the remainder of this text, we shall use the term “node” instead of sensor, to emphasize the network aspects. We also assume that the sensors are able to communicate with nearby sensors. If a sensor can measure its distance to at least two neighbors, it can calculate its position relative to these two nodes by triangulation. However, there will be an ambiguity, in that the position may be mirrored with respect to the line connecting the two other sensors. With a third neighbor, this ambiguity can be resolved. In theory it should be possible to determine the relative position of all sensors. In larger networks, this approach will not be straightforward. If the sensors are placed at random positions, there will always be sensors that are (almost) on top of each other, so the triangulation will not be accurate at all. Furthermore, the distance determinations may suffer from random measurement errors. At the same time, with random networks there will also be many sensors that “see” much more than the required four sensors, so they may be able to improve their position estimates.

In many applications it is not necessary to obtain an *exact* absolute map of the network. If a network is used to monitor an area it may be sufficient to establish which sensors are near to each other, and which are more distant. The scale of the map may not be relevant, although an indication can be obtained, i.e.

from the average sensor density. The result can be improved by including beacons that can establish their position, e.g. with gps. However, this will not be considered here. We limit our study to networks of identical sensors that are left to themselves once they have been delivered, and establish how well they can determine the network structure.

A simple way to determine the distance between different nodes in the network would be, to count the minimum number of hops that are needed to get a signal from one sensor to another. We assume that the signals travel with the speed of light (so no sound signals), and the delay between receiving and broadcasting the signal is constant. In reality, communication between hundreds of thousand of sensors may be a problem. But, when only one node generates a signal, which is then forwarded by the other nodes, the first time that it arrives at any of the other nodes determines the shortest path and therefore the distance between them. We shall show in the remainder of this paper, that a measure of the distance between nodes can be obtained from the number of hops that separates these nodes. We shall consider homogeneous distributions with a variety of sensor densities and also consider the effects of inhomogeneities in the network. Using this distance measure a map of the network can be obtained, but possibly with dispersion.

## 2 Methods

In all our simulations we shall consider a network of  $N$  nodes distributed randomly within some area  $A$ . This leads to a node-density:  $\rho = N/A$ . Unless stated otherwise, nodes are distributed homogeneously over a square area of unit size. This is not a serious restriction since any square area can be normalized to unity by selecting an appropriate length scale. The positions of all  $N$  nodes are known in the simulation. But, we will consider what information can be derived from the number of “hops” that separate any two nodes in the system, i.e. the hop-distance.

First we define a hop-range  $R$ . This is the maximum range at which nodes can contact each other. Two nodes in the network are neighbors if the distance between them is less than the hop-range. Next, we define a hop-route between two end-nodes as a series of neighboring nodes that begins at one end-node and ends at the other. Then, a shortest hop-route between two nodes is defined as the hop-route with the least number of nodes, and finally the hop-distance is the number of hops in a shortest hop-route. In some cases it may not be possible to find any hop-route between the two nodes. In that case the hop-distance is undefined. More than one shortest hop-route may be possible between two nodes, but the hop-distance is unique.

We are interested in the relation between the hop-distance and the geometrical distance under a variety of circumstances. In an actual network, hop-distances are determined by re-broadcasting messages that arrive from neighboring sensors. In essence the network performs massive parallel computing. However, when using a computer to calculate the shortest path considerable computational resources, especially when the network is large and averaging over a number of network realizations is required. To circumvent this problem, we use an alternative approximation of the hop distance. This is done by starting in one of the end-nodes and considering all neighbors and selecting the one that is nearest to the other end-node. This node can then be added to the hop-route, and the search can be continued from there.

In some cases, it may not be possible to construct a directed-hop-route in this way. But, if the node-density is not too low this procedure typically produces a route and a hop-distance that is near to the shortest hop-route, but with much less computational effort. In principle we will use the directed search as a faster algorithm to approximate the hop distance for large networks. However, we shall also compare extensive and directed search results since this may provide additional information about the shortest hop-route under various circumstances.

In the limit of infinite density, the hop-distance between two nodes is equal to the actual distance divided by the hop-range, rounded up to the nearest integer. At finite density, the hop-distance overestimates the actual distance because the shortest hop-route is not straight, and the separation between neighboring nodes is less than the hop-range. If this overestimate is constant it can be corrected for with a calibration procedure and a correct distance estimate can be obtained even at relatively low densities.

In general, such a calibration will have to be done numerically. However, at sufficiently high densities a first order correction can be performed analytically, at least for the directed search. For this, consider that the number of nodes in some area  $A$ , sufficiently smaller than the overall network, follows a Poisson distribution. In particular, the probability that the area is empty equals  $\exp(-\rho A)$ , where  $\rho$  is the density of nodes.

During a directed search, each next node will be shifted towards the end-node, with respect to the previous node, by an amount slightly less than the hop-range  $R$ . This shift deficit ( $\delta$ ), determines the distance correction. It is not constant but distributed around some average. The shift deficit at any particular node will be more than  $\delta$  if the hop-range around the previous node is empty between  $R-\delta$  and  $R$ . Thus, it depends on the area:

$$A(\delta) = \int_{R-\delta}^R 2\sqrt{R^2 - x^2} dx \approx \int_0^\delta 2\sqrt{2Ry} dy = \frac{4}{3}\sqrt{2R}\delta^{3/2} \quad (1)$$

The approximation in this result is valid if the density is sufficiently high so only the values  $\delta \ll R$  will be relevant. The probability that the aforementioned area is empty is determined from the Poisson distribution. From this we derive the probability density for  $\delta$  and its expectation value:

$$\begin{aligned} \rho(\delta) &= -\frac{d}{d\delta} e^{-\rho A(\delta)} \approx \frac{4}{3}\rho\sqrt{2R}\delta^{1/2} \exp\left(-\frac{4}{3}\rho\sqrt{2R}\delta^{3/2}\right) \\ \langle \delta \rangle &\approx \int_0^\infty \rho\sqrt{2R}\delta^{3/2} \exp\left(-\frac{4}{3}\rho\sqrt{2R}\delta^{3/2}\right) d\delta = \frac{R\Gamma(5/3)}{\left(\frac{4}{3}\sqrt{2}\rho R^2\right)^{2/3}} = \frac{1.27R}{(\pi\rho R^2)^{2/3}} \end{aligned} \quad (2)$$

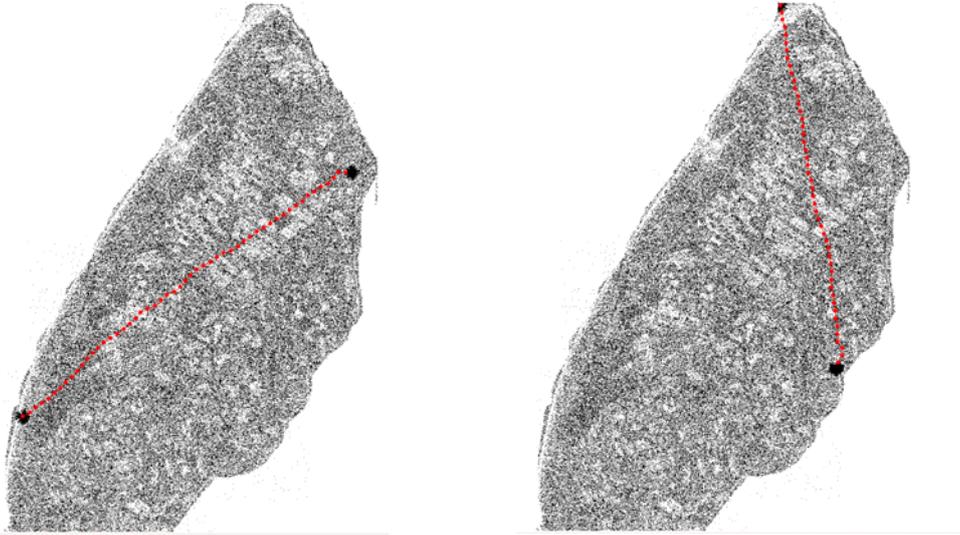
In the last integral, we have taken the integral to infinity. Once more, this provides a valid approximation if the density is sufficiently high, so that only small values of  $\delta$  contribute to the integral. The average shift reduction  $\langle \delta \rangle$  depends on the hop-range and is reduced by the average number of nodes ( $\pi\rho R^2$ ) within the hop-range. For example, if  $\pi\rho R^2 = 100$ ,  $\langle \delta \rangle = 0.059R$ , in that case the distance is overestimated by approximately 6%. In general, the distance ( $d$ ) can be estimated from the hops-distance ( $n$ ) by multiplying it with the hop-range minus the average shift reduction:

$$\langle d \rangle = n(R - \langle \delta \rangle) = nR\left(1 - \frac{1.27}{(\pi\rho R^2)^{2/3}}\right) \quad (3)$$

### 3 Results

A typical result of our simulations is shown in Figure 1. The grey scale map of an existing island was used as a density function to randomly distribute 200000 sensor nodes. In this way, a natural variation of node density over a large area could be reproduced. The two figures show shortest hop-routes that have been determined between selected nodes. What can be observed is that the two routes meander slightly around the shortest (straight) geometrical route. However, the deviation is small in this case and the number of hops provides a good estimate of the actual geometrical distance. This result does not appear to be affected by the density fluctuations in the network. In fact, the distance between neighboring nodes in each of the routes is relatively constant, also in areas with much lower node density.

In order to determine whether the accuracy of the distance estimates in our simulations is always correct, we have considered a number of networks with randomly distributed nodes and numerically obtained distributions for the hop-distance distributions by extensive and directed search ( $n_e$  and  $n_d$  respectively). We also need to consider how the distribution of hop-distances depends on the geometric distance. In principle this distribution depends on three parameters: the geometric distance  $d$  between the nodes, the density ( $\rho$ ) of the nodes and the (maximum) hop-range ( $R$ ). However, only two of these parameters have to be considered since the system is scale-invariant. For instance, the hop-distance distributions can be expressed as a function of normalized geometric distance  $d/R$  and normalized density ( $\pi\rho R^2$ ).



*Figure 1: Shortest hop routes on a simulated network of 200000 inhomogeneously distributed sensor nodes on the island of Texel. The node density is inhomogeneous according to the terrain features that were determined from a gray scale map of the island. The red dots indicate nodes that form a shortest path between two selected nodes. (The individual nodes may not show up very well in print.) To indicate the node-density, all neighbors that lie within 1 hop distance ( $R = 0.015$ ) from the start- and end node are connected to these nodes by small black lines.*

The results of our simulations are shown in table 1. In all cases the hop-distance was determined for twelve distinct node-pairs and the average was determined as well as the standard-deviation. The normalized geometric distance was varied from 3 to 100 hops. The normalized node density was varied from 10 to 100. A normalized node density of 3 was also attempted but almost never produced a shortest hop-route. Even at a normalized node density of 10 the directed search algorithm often produced an undefined distance.

In all cases, the hop-distance overestimates the normalized geometric distance. The discrepancy varies from 5-7% at the highest density to 30-40% at the lowest density. Even larger discrepancy is observed at normalized geometric distances of 3 and 10 hops, but this is a round-off error due to the fact that the hop-distance is an integer. Important, however is that the discrepancies are rather consistent so that considerable correction should be possible. The standard deviation in the hop-distance is always less than 10% for a normalized geometric distance of 10 hops, i.e. of the same order as the round-off error, and follow the square-root-of-n-law for random numbers, decreasing to 2% or less for a normalized geometric distance of 100 hops. With a good correction it should be possible to obtain distance estimates with 2-3% accuracy at distances of 20 hops and more.

We find no indication that the relative discrepancy between hop- and geometric distance depends on the number of hops. Apparently, non-linearity and correlation between overlapping environments of neighboring nodes do not seem to affect the amount of overestimation. Thus, the applicability to estimate the geometrical distance for short routes is limited only by the round-off errors that occur due to the integer number of steps in a route.

As stated above, the discrepancy does increase considerably with decreasing normalized node density. However, since the standard deviation is limited to 1-2 steps in each case it is always possible apply a correction factor. In fact, our theoretical estimation (see equation (3)) only slightly underestimates the discrepancy and predicts the hop-distance with an accuracy better than 1% in all cases. In view of the standard deviations in the obtained hop-distances, this should usually be sufficient. If needed a higher-order correction should provide even better results.

Table 1: Hop-distance  $n_e$  and  $n_d$  (extensive resp. directed search) for various normalized geometric distances between the nodes ( $d/R$ ) and node density ( $\pi\rho R^2$ ). In each case, we show the average and the standard deviation for twelve simulated case (see text).

$\pi\rho R^2$	10				30				100			
$\downarrow d/R$	$n_d$	$\sigma n_d$	$n_e$	$\sigma n_e$	$n_d$	$\sigma n_d$	$n_e$	$\sigma n_e$	$n_d$	$\sigma n_d$	$n_e$	$\sigma n_e$
100	138.5*	2.1	130.1	1.7	115.7	1.2	112.5	0.8	106.8	0.7	105.4	0.5
30	41.8*	1.6	39.4	1.5	35.0	0.7	34.2	0.4	32.3	0.5	32.0	0.0
10	14.4	1.3	14.0	0.9	11.8	0.6	11.8	0.6	11.0	0.0	11.0	0.0
3	4.9	0.6	4.9	0.5	4.0	0.0	4.0	0.0	4.0	0.0	4.0	0.0

\* undefined distance in 60-80% of all cases

The results of the simulations shown in Table 1 were obtained with a homogeneous network. For practical applications, this is not sufficient. In a featured terrain, any distribution of sensor nodes will suffer from inhomogeneities in density and in the range at which neighbors can be contacted (hop-range). Density variations may lead to variable overestimates of the local distance estimates. However, this becomes a problem only if the density variations are very large or if the mean (overall) density is relatively small. Our results in Figure 1 and other unpublished simulations indicate that minor density fluctuations during parts of the route between the nodes have little effect on the estimated distance between them.

A possibly more serious factor is formed by hop-range variations since these directly scale distance measurements. Simulations where the hop-range in one half of the area is a factor two larger than in the other showed a kink in the shortest hop-route between nodes on either side of the interface (see Figure 2). This type of kinks may lead to considerable errors in the distance estimates when the hop-range varies in the area that is covered by the network.

## 4 Discussion

We have investigated distance estimation in a network of sensor nodes that are only able to establish contact with nearby neighbors. The hop-distance (the number of communication ‘‘hops’’ that is necessary for a message to travel from one node to another) is taken as an estimate of the distance between them. Using a pentium 3 GHz computer it was possible to numerically evaluate networks of more than 300000 nodes and extensive search to determine the shortest route.

For normalized densities of 10 and distances of 10-30 hops or more the standard deviation between hop-distances for the same geometric distance is no more than a few percent. The hop-distance overestimates the geometric distance, but this can be resolved with the simple correction provided in equation (3). At shorter distances a much larger round-off error occurs due to the integer nature of the hop-distance. To determine a global map of the network such short distances should not be used. In fact, for the large networks considered in the study, the hop-distance may be quite sufficient to estimate geometric distances and perform almost as well as explicit geometric distance measurements between neighboring sensors. This favors the development of simple sensors that use the hop-distance and do not measure distances explicitly.

Inhomogeneity needs to be considered when a large number of sensors is dropped in a terrain such as illustrated in figure 1. We considered the types of inhomogeneity that are to be expected in a featured terrain. Certainly it will not be possible to distribute the sensors evenly. However, our results indicate that this has little effect on the estimated hop-distances as long as the minimum (local) density is sufficient. Variations of hop-range should also be expected since difference in terrain structure and vegetation will affect the ability of sensors to communicate. As illustrated in figure 2 this type of inhomogeneity leads to

deformation of the shortest hop-route between two nodes. However, the route is identical to that of a refracted optical light ray. This analogy was used in previous studies to determine optimal transfer routes [6]. It may be possible to use the same analogy to detect hop-range inhomogeneities and correct for them. It may also be necessary to consider a three dimensional landscape.

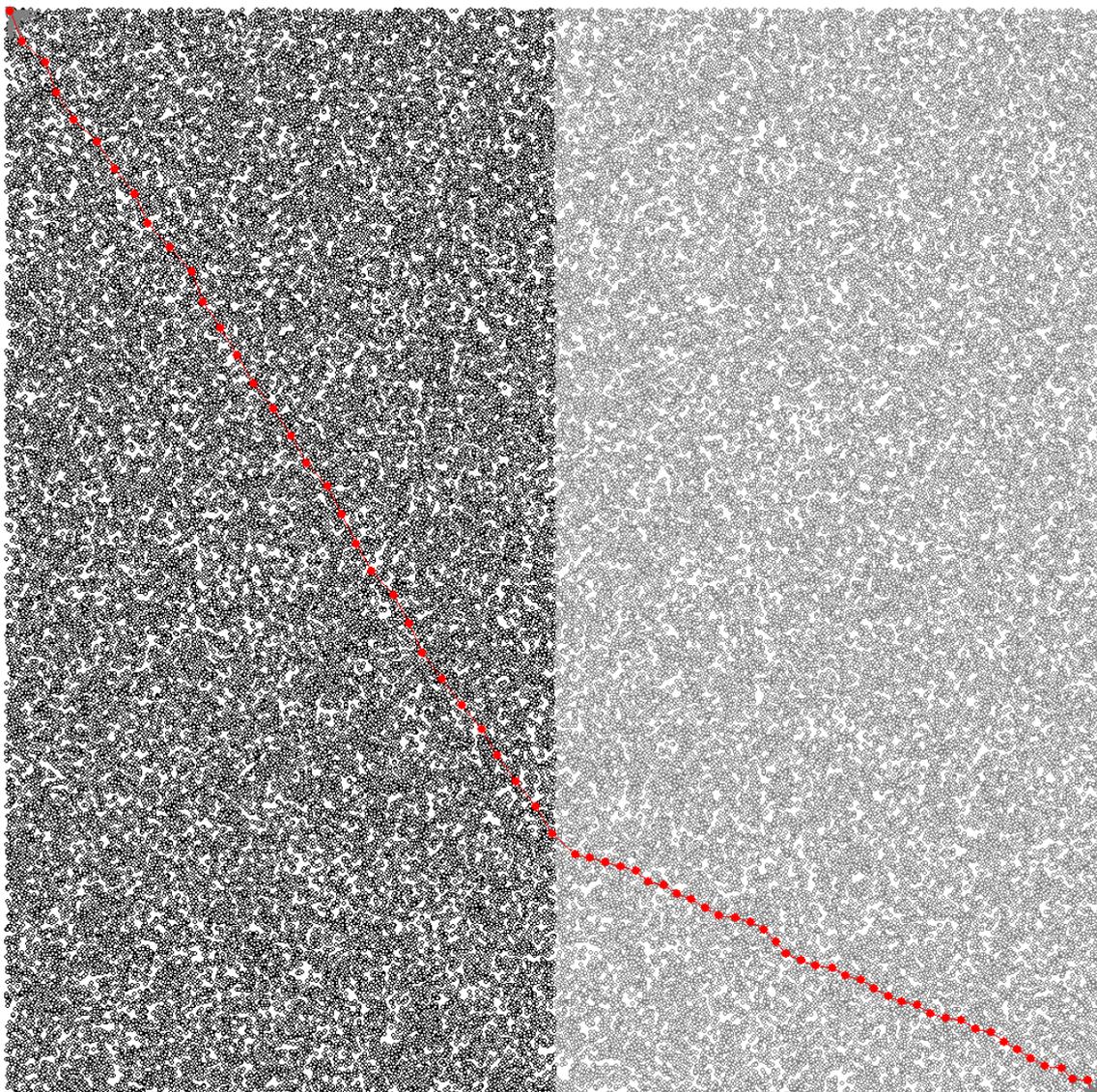


Figure 2. Simulation of a shortest hop-rout in a network of 200000 sensor nodes are distributed randomly over a certain area. At the left side, the communication range is twice as large as that of the right side (this is reflected in the distances between nodes). The shortest path, measured in number of hops, follows a law similar to Snell's law [6].

The densities used in our scenarios are relatively high, with normalized densities between 10-100. This implies that each node can communicate with 10-100 neighbors that are within the hop-range. For normalized densities significantly below 10, it is often not possible to establish a hop-route. Thus, we find that the shortest hop-route can be used as a distance measure for large network ( $> 10^4$  nodes) with relatively high normalized density ( $> 10$ ). Such a type of network could be characterized as "smart dust". [7]. It can be considered if sufficiently cheap sensors are available. What may considered cheap depends on various factors such as the life-span of the network en the type of application that it is used for.

A “smart dust” type of network would be fault tolerant since each sensor has a number of neighbors to communicate with. Even if a small fraction of the sensors cannot connect to the network, the functionality of the network as a whole is not at risk. The sensors would probably function with infrequent communication where every now and then, one node would send a message which is re-broadcasted by the others, which then also know their distance from the broadcasting node. A few of these distances will be sufficient for each node to determine its global position within the network.

In summary, we find that the number of re-broadcasts required to transmit a message between non-neighboring sensors in a network can be used as a simple tool to estimate geometrical distances between them. This approach works in large networks ( $\geq 10^4$  nodes) with relatively high density ( $\geq 10$  sensors within the hop range). This indicates that the approach is suitable for large, high-density networks of low-cost sensors. Such networks may be characterized as “smart dust”. Inhomogeneous hop-range due to vegetation and other obstacles are a cause of error that is yet to be avoided. But, density fluctuations that result from delivery in a featured terrain do not seem to affect distance estimates.

## References

- [1] S. Dulman, A. Baggio, P. Havinga and K. Langendoen. A Geometrical Perspective on Localization. Proceedings of the 1st ACM Int. Workshop on Mobile Entity Localization and Tracking in GPS-less Environments, 85-90, 2008.
- [2] L. Hu and D. Evans. Localization for Mobile Sensor Networks. Proceedings of the International Conference on Mobile Computing and Networking (MobiCom), 2004.
- [3] R. Nagpal, H.E. Shrobe and J. Bachrach: Organizing a Global Coordinate System from Local Information on an Ad Hoc Sensor Network. Proceedings of the 2nd International Symposium on Information Processing in Sensor Networks (IPSN) 2003: 333-348.
- [4] S.Čapkun , M. Hamdi, and J.-P. Hubaux. GPS-free positioning in mobile Ad-Hoc networks. Proceedings of the IEEE Hawaii Conference on System Sciences, 255-264, 2001.
- [5] Th.M. Hupkens and O.J.G. Somsen. Self-Localization of Sensors in Networks of Randomly Distributed Sensors, Proceedings of the 1st International Society Conference of Military Sciences, Submitted, 2009.
- [6] R. Catanuto, S. Toumpis and G. Morabito. On asymptotically optimal routing in large wireless networks and Geometrical Optics analogy. Computer networks 53: 1939-1955.2009
- [7] A. Boukerche, I. Chatzigiannakis, S. Nikoletsemb. A new energy efficient and fault-tolerant protocol for data propagation in smart dust networks using varying transmission range. Computer communications 29: 477-489. 2006