

Reasoning About Multi-Attribute Preferences*

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Introduction Many preference models are quantitative. These kinds of models are based on utilities; a utility function determines for each outcome a numerical value of desirability. However, it is difficult to elicit such models from users. Humans generally express their preferences in a more qualitative way. Therefore, qualitative preference models would provide a better correspondence with human cognitive representations.

One of the binary preferences in [3] expresses that any state where φ is true is strictly better than any state where ψ is true. If applied to object names, it can express that object i is preferred over object j . The preference expressed in this way is a very strong kind of preference. It requires that all of i 's relevant properties are considered more important than j 's properties. The motivation to introduce multi-attribute preference logic is to enable the specification of principles that allow to derive preferences over objects from their properties in a weaker sense. In most qualitative preference orderings it is possible that i has at least one property that is considered more important than a property that j has but j is still preferred over i . Moreover, the logic should provide the means to derive a preference of one object over another from a specification of the properties of these objects and an importance ranking associated with these properties.

[1, 2] have defined various orderings (called $\#$, \top and κ ordering) to obtain object preferences from a property ranking, which indicates the relative importance or priority of each property. These orderings are explained below. The advantage of defining preference orderings in a logic instead of providing set-theoretical definitions is that it formalizes the reasoning about object preferences. From a practical point of view, the logic allows to provide rigorous formal proofs for object preferences derived from property rankings. From a theoretical point of view, it provides the tools to reason about preference orderings and allows, for example, to prove that whenever an object is preferred over another by the \top ordering it also is preferred by the $\#$ ordering.

Preference orderings For the $\#$ ordering, first consider the most important property. If some object has that property and another does not, then the first is preferred over the second. If two objects both have the property or if neither of them has it, the next property is considered.

For the \top ordering, consider the highest ranked or most important property that is satisfied. If that property of one object is ranked higher than that of another object, then the first object is preferred over the second. If those properties are equally ranked, then both objects are equally preferred.

For the κ ordering, consider the most important property that is not satisfied. If that property of one object is less important than the property of another object, then the first object is preferred over the second. If those properties are equally important, then both objects are equally preferred.

Multi-attribute preference logic We propose a modal logic that extends binary preference logic as presented in [3] with names for objects and a modal operator to characterize properties or attributes. This language extension allows us to talk about properties, objects and associated preferences explicitly.

The basic concepts in the semantics for MPL are *objects* and *properties* those objects may have. This is visualized in Figure 1. Properties are naturally represented by sets of worlds in modal semantics, which we call *clusters*. As we want to use properties to classify the ranking of objects, properties are ordered in correspondence with their relative importance; such an order is called a *property ranking*.

Objects are identified with particular sets of worlds. The idea is that we can derive the properties of an object from the worlds which define the object. To ensure that objects are coherent, that is, have a uniquely

*This is an abstract of [4]. More information about the ideas in this abstract and references to relevant literature can be found there.

defined set of properties, the worlds that define the object need to be *copies* of each other. A world is a copy of another world if it assigns the same truth values to propositional atoms. The general idea is that the worlds that constitute an object act as representatives for that object in a certain property cluster. So for each object, there is a world for each property in the property ranking that the object has.

The language of MPL consists of a propositional part (with standard syntax and semantics) and several modal operators. Object names (e.g. i, j) are nullary modal operators that are only true in worlds belonging to one and the same object. \Box^{\leq} , $\Box^{<}$ and $\Box^=$ (with duals \Diamond^{\leq} , $\Diamond^{<}$ and $\Diamond^=$) are standard modal operators with associated accessibility relations derived from a given total preorder on worlds. U (with dual E) is the global modal operator. $\Box^{\#}$ (with dual $\Diamond^{\#}$) inspects all worlds that are not equally ranked as the current world, and moreover, that do not have copies that are equally ranked as the current world.

A cluster, which is a set of worlds that are ranked equally, is said to characterize a property φ (denoted $C(\varphi)$) if φ is true in all objects that are represented in that cluster, and all objects that satisfy φ are represented in that cluster; $C(\varphi)$ is defined as $\Box^= \varphi \wedge \Box^{\#} \neg \varphi$. The operator C allows us to express a property ranking in MPL. In the model in Figure 1, the clusters characterize three desired properties of houses, and a fourth property of not having any of the three desired properties.

All preference orderings above can be defined in multi-attribute preference logic in such a way that they are equivalent to those of [1]. The definitions are displayed in Figure 2. For example, the definition of the $\#$ ordering states that an object i is preferred over j if there is a world that belongs to object i , there is no world belonging to object j in the same cluster, and for all more important clusters, either both i and j are represented in that cluster or neither is. Without going into detail, it can be seen that when an object is preferred to another according to the κ ordering, it is also preferred according to the $\#$ ordering.

A ranked knowledge base, which is Brewka's version of a property ranking, can be translated into multi-attribute preference logic in such a way that every multi-attribute preference model that is a model of the translation yields the same preference ordering as the original ranked knowledge base.

Future work Issues to explore further include dependencies between attributes, incomplete information and preferences, and preference change. We will also focus on argument-based reasoning. This is a natural way to reason, and it might be better equipped to deal with inconsistent, incomplete or changing beliefs and preferences. Moreover, it provides a computational approach to derive preferences. Eventually, our aim is to integrate an expressive preference language into a larger negotiation framework. This framework will also contain associated strategies for negotiation and will be the core of a negotiation support system.

References

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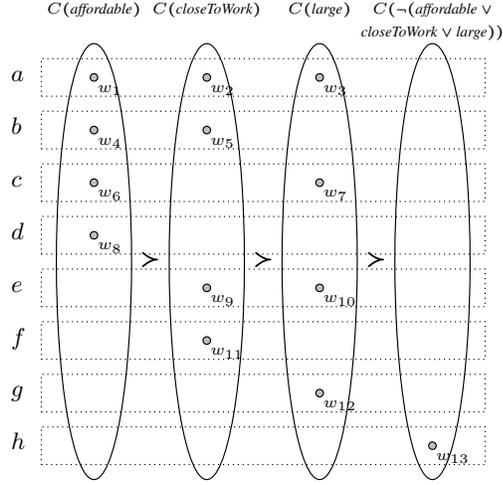


Figure 1: Visualization of an MPL model

$$\begin{aligned}
 \text{pref}^{\bar{}}(i, j) &::= E(i \wedge \neg \Diamond^= j \wedge \Box^{<} (\neg i \wedge \neg j)) \\
 \text{pref}^c(i, j) &::= E(i \wedge \neg \Diamond^= j \wedge \Box^{<} (\Diamond^= i \wedge \Diamond^= j)) \\
 \text{pref}^{\#}(i, j) &::= E(i \wedge \neg \Diamond^= j \wedge \Box^{<} (\Diamond^= i \leftrightarrow \Diamond^= j))
 \end{aligned}$$

Figure 2: Preference ordering definitions