Introduction: Business Information Systems, Transition Systems, and Petri Nets

prof.dr.ir. Wil van der Aalst
www.vdaalst.com
• Lecturers:
  - prof.dr.ir. Wil van der Aalst (lectures)
  - dr.ir. Boudewijn van Dongen (instructions & assignment)
  - dr. Dirk Fahland (instructions & assignment)

• Lectures: There are two times two hours of lectures every week: Monday 1\textsuperscript{st}+2\textsuperscript{nd} hour (8.45-10.30) and on Thursday 5\textsuperscript{th} and 6\textsuperscript{th} hour (13.45-15.30) in AUD 7.

• Instructions: There are four hours of instructions each week: on Friday 5\textsuperscript{th}+6\textsuperscript{th}+7\textsuperscript{th}+8\textsuperscript{th} hour (13.45-17.30). There are two instruction groups: one in MetaForum room 06 and one in MetaForum room 07.
Practicalities 2/7 (Examination)

• The final mark will be based on:
  – A “pre-exam” focusing on classical Petri nets (1 point). This written exam is scheduled on Friday 17-5-2013 from 14.00-15.30 (as part of the regular BIS instruction).
  – A “CPN assignment” (3 points) where a large CPN model is constructed and analyzed. There are two deadlines: 6-6-2013 (Part I) and 23-6-2013 (Part II).
  – A “final exam” (6 points). This written exam is scheduled on Thursday 4-7-2013 from 9.00-12.00.

• The “pre-exam” and “CPN assignment” are mandatory and will expire after the first “final exam” on 4-7-2013.
• The exam in the interim period (15-8-2013) will cover all material and therefore much more difficult to pass.
• Students are advised to not take any risk, and pass the first time (to avoid redoing the entire course).
Practicalities 3/7 (OASE and CPN Tools)

- **OASE** is used to distribute material, etc. Register via OASE before the course starts (**mandatory**).

- **CPN Tools**
  - Plays a prominent role in this course.
  - Download the software via [http://cpntools.org/](http://cpntools.org/) (Version 3.5.7). Install the right version, do it today!
  - See online documentation (including pointers to SML).

- Additional tools used: ProM 6.2 (and possibly tools to draw classical Petri nets, WF-nets, and BPMN).
• **Material**
  - Slides (via OASE).
  - Exercise bundle (via OASE).
  - Supplementary papers (via OASE).
• Information about the book can be found on http://cpntools.org/books/modeling.
• The book can be ordered via the publisher's Web page or via Amazon, BOL, selexyz, etc.
• Get it today if you do not have it already!
## Practicalities 5/7 (Planning)

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<td>22-4-2013</td>
<td>Lect.</td>
<td>Introduction, transition systems, Petri nets (1)</td>
<td>Read Chapters 1-3 of book.</td>
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<td>25-4-2013</td>
<td>Lect.</td>
<td>Petri nets (2)</td>
<td>Read Chapter 3 of book.</td>
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<td>26-4-2013</td>
<td>Inst.</td>
<td>Transition systems, Petri nets</td>
<td>Make all exercises in Section 1 and part of the exercises in Section 2.</td>
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<td>3-5-2013</td>
<td>Inst.</td>
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<td>17-5-2013</td>
<td>Exam.</td>
<td>Pre-exam focusing on classical Petri nets (1 point)</td>
<td>Study Chapters 1-4 and all exercises in Sections 1-2.</td>
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<td>Inst.</td>
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<td>Simulation (9)</td>
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<td>Reachability Analysis and basic properties (10)</td>
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<td>6-6-2013</td>
<td>Ass.</td>
<td>Deadline Part I of “CPN assignment”</td>
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<td>Final exam (6 points)</td>
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Practicalities 6/7 (Assignment)

• The CPN assignment will be distributed separately.
• The assignment will be handed out and explained during the instruction in 17-5-2013 (immediately following the pre-exam).
• The assignment consists of two parts.
  – Part I needs to be handed in on 6-6-2013 (23.59).
  – Part II needs to be handed in on 23-6-2013 (23.59).
• Warning: extensive checks for plagiarism!
• The points for the assignment are only valid in conjunction with the first exam.
Practicalities 7/7 (Warnings)

- This course is much more difficult than it seems! Understanding a model easy, making a model is difficult (see statistics of earlier years).
- Examples in book are relatively easy compared to exam questions. Several old exams have been posted already.
- Start working on the assignment ASAP to avoid an unbalanced workload over the quarter.
- Points for "pre-exam" and "CPN assignment" expire. Only one 6 ECTS exam attempt in 2013/2014 (due to changes in the curriculum). Not passing now will generate extra work and potentially create serious problems!
Become a top cook rather than a wannabe food critic!
On the role of processes
What do these situations have in common?

(business) process supported by an information system
On the relation between processes, systems, and models.
Use of process models

- Insight
  for a better understanding of the system

- Analysis
  validation and verification

- Realization
  specification or configuration of an information system
Transition Systems
Dynamic systems: discrete vs continuous

- Y-axis is state space
- Transition relation is not described here
Basic model: Transition systems

• Any discrete dynamic system can be described as a transition system

• Low level model: "Mother of all (process) models"

---

Initial state:
- idle
- card
- pin
- balance
- money
- offer
- choice
- violation
- payout
- output_card

State transitions:
- From idle to card
- From card to pin
- From pin to balance
- From balance to money
- From money to offer
- From offer to choice
- From choice to violation
- From violation to payout
- From payout to output_card

Transition states:
- state
- transition
A transition system is a triple \((S, TR, s_0)\) where

- \(S\) is the finite state space,
- \(TR \subseteq S \times S\) is the transition relation,
- \(s_0\) is the initial state.

The elements of \(S\) are states.
The elements of \(TR\) are transitions.
Example: Elevator

- Define transition system
- Draw state-transition diagram.

- $S = \{0, 1, 2, 3, 4\}$
- $TR = \{(0, 1), (1, 2), (2, 3), (3, 4), (4, 3), (3, 2), (2, 1), (1, 0)\}$
- $s_0 = 0$
State explosion problem

50 states

$50^{15} = +/- 30,517,578,125,000,000,000,000,000$ states
Large state spaces ...

communication protocol \((10^6\) states\)
link layer of the IEEE1394 standard (FireWire) simulated using two communication nodes and a bus (25,000 states)
Exercise 1.12 We consider the problem of the dining philosophers as introduced by Dijkstra: “Five philosophers, numbered from 0 through 4 are living in a house where the table laid for them, each philosopher having his own place at the table: Their only problem besides those of philosophy is that the dish served is a very difficult kind of spaghetti, that has to be eaten with two forks. There are two forks next to each plate, so that presents no difficulty: as a consequence, however, no two neighbours may be eating simultaneously.” Figure 3 illustrates this example.

1. Model the philosophers as a transition system. It is assumed that each philosopher simultaneously (and indivisibly) picks up his pair of forks. Analogously, he puts them down in a single indivisible action. Each philosopher may be in two states think and eat.

2. Does the model have terminal states (i.e., deadlocks)?

Exercise 1.13 Figure 4 depicts a simplified remote control of a TV. It has six buttons to choose a channel, one button to regulate the volume, one button to mute the sound (and to turn it on again), and one button to switch the TV off. We consider the remote control and the corresponding TV as a system and assume that the possible states of this system are controlled by the buttons on the remote control.

1. Describe all possible states of this system.

2. The transition relation is too large to be depicted as a state-transition diagram. Give therefore examples of possible and impossible transitions. You should especially pay attention to switching the TV on and to the use of the volume button in combination with the mute button.
Relation to data modeling and systems development
Process models versus object/data models

system model = process model + data/object model
Example of a data/object model

UML class diagram

Customer
- Name
- Number
- Age

Sale
- date
- discount
- price

Sales person
- number
- experience
- authorization

Product
- number
- weight
- in stock?
Other data/object modeling techniques

Entity-Relationship (ER) diagrams

Crow’s Foot diagrams

NIAM/ORM diagrams
Business process models versus information system models

“world” business process models, analyzes information system supports analyzes, specifies validates

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<th>information system model</th>
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<tbody>
<tr>
<td>validates models, analyzes</td>
<td>implements, configures analyzes, specifies</td>
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<tr>
<td>supports</td>
<td>controls</td>
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system model

data model

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<th>Patient</th>
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process model

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Example of a process model

A Petri net modeling order processing
Other process modeling techniques

• UML activity/statechart diagrams
• Business Process Modeling Notation (BPMN)
• Event-driven Process Chains (EPCs)
• IDEF/DFD diagrams
• BPEL
• …
# Tool support

## Business process modeling tools

- Protos
- ARIS
- BPM|one
- IBM WebSphere

## Model-based analysis tools

- Simula
- Arena
- IBM WebSphere
- ProM
- Woflan
- CPN Tools

## Business process enactment tools

- BPM|one
- SAP ERP
- Oracle’s JD Edwards
- IBM WebSphere

## Tools for analyzing running business process

- ARIS
- BPM|one
- IBM WebSphere
- ProM
- Fluxicon’s Disco
Lifecycle of Business Information Systems

- Requirements
- (Re)design
- Design analysis
- Implementation
- Production
- Distribution
- Deployment
- Runtime analysis
- Monitoring
- Execution
- Adjustment
- Configuration
- Migration
Trends in information systems development over past decades

Trend 1: From data orientation to process orientation

Trend 2: From programming to assembling

Trend 3: From programming to configuration

Trend 4: Toward redesign and organic growth
Generic functionality is isolated in separate components often driven by "models"
Observations

• Process models are everywhere.
• Must be correct, precise, and unambiguous.
• Modeling and analysis techniques are indispensable.

Goals:

• Modeling and analysis techniques for insights, actionable information, process management, configuration, and implementation.
• Information systems and tools supporting the complete lifecycle.
Petri nets
Process modeling

- Emphasis on **dynamic behavior** rather than structuring the state space.
- Transition systems are too low-level.
- Start with **classical Petri nets**.
- Later the basic model is extended with:
  - **Color**
  - **Time**
  - **Hierarchy**
Why Petri nets and not some industrial tool?

• Industrial languages/tools tend to come and go.
• Focus would be on syntactical elements rather than basic concepts.
• Often no formal semantics.
• Only few analysis techniques available.

Goal:
• Teach **foundational concepts** of process modeling and analysis.
Strong point of Petri nets

- Math. foundation
- Graphical notation
- Compactness
- Concurrency, locality
- Analysis techniques
- Tool support
A glimpse of history

Start of concurrency research

Focus mainly on theory

Focus now also on tooling and applications

"Hidden" in many diagramming techniques, e.g., BPMN, EPCs, UML, etc.

1962

PhD thesis C.A. Petri

1970

1980

1990

2000

2010
An informal introduction to Petri nets
Elements of a Petri net

Legend:

- **Place**
- **Transition**
- **Arc**
- **Token**

- **Transition label**
- **Place label**
Syntactic rules

- Directed graph with two types of nodes: places and transitions
- Places may hold zero or more tokens
- State (marking) is distribution of tokens over places
Transition enabling

• A transition is enabled if each of its input places contains at least one token.
Transition firing

- An enabled transition can fire (i.e., it occurs)
- When it fires it consumes a token from each input place and produces a token into each output place
Play “Token Game”

- In the new state, make_photo is enabled
- It will fire, etc.
Remarks

Network structure is static

Atomicity

Interleaving semantics

Max. firing semantics
Non-determinism

Two transitions are enabled but only one can fire.

transition
move23 fires
floor4
floor3
floor2
floor1
floor0
move43
move34
move32
move23
move21
move12
move10
move01

transition
move23 fires
floor4
floor3
floor2
floor1
floor0
move43
move34
move32
move23
move21
move12
move10
move01
Example: Single traffic light
Two traffic lights
Problem
How to make them alternate?
WARNING

It is not sufficient to understand the (process) models. You have to be able to design them yourself!
A more formal introduction to Petri nets
A Petri net is a triple \((P, T, F)\), where

- \(P\) is a finite set of places,
- \(T\) is a finite set of transitions,
- \(F \subseteq (P \times T) \cup (T \times P)\) is a flow relation.

Any diagram can be mapped onto such a triple and vice versa.

\[
P = \{p1, p2, p3\} \\
T = \{t\} \\
F = \{(p1, t), (p2, t), (t, p3)\}
\]
**Intuition:** Define input and output places in terms of the flow relation. Let $t$ be a transition.

- Set $\bullet t = \{p \mid (p, t) \in F\}$ defines all input places of $t$ (the **preset** of $t$)
- Set $t^\bullet = \{p \mid (t, p) \in F\}$ defines all output places of $t$ (the **postset** of $t$)
Determine the pre- and postsets
Multisets

Set

- order of elements does not matter
- same element appears only once

Multiset

- order of elements does not matter
- same element may appear multiple times
Calculating with multisets

\[ \begin{align*}
\text{Pens} & \quad \text{Coins} \quad \text{Buttons} \\
\quad + \quad \quad \quad \quad \quad \quad \quad + \\
\quad - \quad \quad \quad \quad \quad \quad \quad - \\
\quad = \quad \quad \quad \quad \quad \quad \quad \text{undefined}
\end{align*} \]
A **marking** of a Petri net \((P, T, F)\) is a function

\[ m: P \rightarrow \mathbb{N} \]

assigning to each place \(p \in P\) the number \(m(p)\) of tokens at this place. The set \(M\) of all markings of this net is the set of all such functions.

- \(m\) is a *multiset*

\[
\begin{align*}
m(p1) &= 1 \\
m(p2) &= 2 \\
m(p3) &= 0
\end{align*}
\]

or \(m = [p1, 2 \cdot p2]\)
Enablingness

In a Petri net $(P, T, F)$, a transition $t \in T$ is **enabled** at marking $m$: $P \rightarrow N$ if for all $p \in \bullet t$, $m(p) > 0$.

![Petri net diagram]

- $m(p1) = 1$
- $m(p2) = 2$
- $m(p3) = 0$

$t$ is enabled at $m$
Which transitions are enabled?
Transition firing (1)

For a Petri net \((P, T, F)\), let
\[
w: (P \times T) \cup (T \times P) \to \{0, 1\}
\]
the weight function with
\[
w((x, y)) = 1 \text{ if } (x, y) \in F \text{ and } w((x, y)) = 0 \text{ if } (x, y) \notin F,
\]
for all \((x, y) \in (P \times T) \cup (T \times P)\).

\[
\begin{align*}
w((p1, t)) &= 1 \\
w((p2, t)) &= 1 \\
w((p3, t)) &= 0 \\
w((t, p1)) &= 0 \\
w((t, p2)) &= 0 \\
w((t, p3)) &= 1
\end{align*}
\]
Transition firing (2)

For a Petri net \((P, T, F)\), let \(w\) be the weight function and let \(m: P \rightarrow \mathbb{N}\) be the current marking.

A transition \(t \in T\) can fire if it is enabled at \(m\). The firing of \(t\) yields a new marking \(m': P \rightarrow \mathbb{N}\) where for all places \(p \in P\),

\[
m'(p) = m(p) - w((p, t)) + w((t, p)).
\]

\[
m'(p1) = 1 - 1 + 0 = 0
\]
\[
m'(p2) = 2 - 1 + 0 = 1
\]
\[
m'(p3) = 0 - 0 + 1 = 1
\]
Which transitions can fire and what are the successor markings?

diagram showing a Petri net with places p1, p2, p3, t1, t2, t3, t4, r1, rg1, g1, go1, o1, r2, rg2, g2, go2, or2, x, y.
A Petri net system \((P, T, F, m_0)\) consists of a Petri net \((P, T, F)\) and a distinguished marking \(m_0\), the initial marking.

\[
P = \{p1, p2, p3\} \\
T = \{t\} \\
F = \{(p1, t), (p2, t), (t, p3)\} \\
m_0(p1) = 1 \\
m_0(p2) = 2 \\
m_0(p3) = 0
\]
Example

- Model a circular railway system with four stations (st1, st2, st3, and st4) and one train.
- At each station passengers may "hop on" or "hop off". This is impossible if the train is moving.
- The train has a capacity of 50 persons; if the train is full, no new passengers may hop on.
- Model the above process in terms of a Petri net.
- What is the number of reachable states?
- Hints:
  - How to describe the state of the train in terms of its location (e.g., moving from st1 to st2) and number of passengers (e.g., 36)?
  - What are possible actions?
  - When are they possible?
Solution
Solution

51 x 4 = 204 states
<table>
<thead>
<tr>
<th>week</th>
<th>date</th>
<th>Type</th>
<th>topic</th>
<th>to prepare</th>
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<tbody>
<tr>
<td>17</td>
<td>22-4-2013</td>
<td>Lect.</td>
<td>Introduction, transition systems, Petri</td>
<td>Read Chapters 1-3 of book.</td>
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<td></td>
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<td>nets (1)</td>
<td>Read Chapter 3 of book.</td>
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<td>25-4-2013</td>
<td>Lect.</td>
<td>Petri nets (2)</td>
<td>Make all exercises in Section 1 and part of the exercises in Section 2.</td>
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<tr>
<td></td>
<td>26-4-2013</td>
<td>Inst.</td>
<td>Transition systems, Petri nets</td>
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<td>18</td>
<td>29-4-2013</td>
<td>Lect.</td>
<td>TU/e closed</td>
<td>Read Chapter 4 of book.</td>
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<td></td>
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<td>Modeling with Petri nets (3)</td>
<td>Make all exercises in Section 2.</td>
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<tr>
<td></td>
<td>2-5-2013</td>
<td>Lect.</td>
<td></td>
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<td>3-5-2013</td>
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<tr>
<td>19</td>
<td>6-5-2013</td>
<td>Lect.</td>
<td>Extending Petri nets with channels</td>
<td>Read Chapter 5 of book.</td>
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**Exercise**

- Week 17: Read Chapters 1-3 of book. Read Chapter 3 of book. Make all exercises in Section 1 and part of the exercises in Section 2.
- Week 18: Read Chapter 4 of book. Make all exercises in Section 2.
After this lesson you should be able to

- Explain the relationship between processes, systems and models.
- Describe a system as a transition system and represent it as a state-transition diagram.
- Explain the relation between data and process models.
- Describe the role of business process models in various classes of tools.
- Explain the ingredients of the network structure of Petri nets and the behavior of Petri nets.
- Model simple systems and processes as Petri nets.
- Draw the accompanying Petri net if it is given as a tuple, and vice versa.
- Fully understand the formal semantics.