More on Classical Petri Nets

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Petri nets

Math. foundation

Graphical notation

Compactness

Concurrency, locality

Analysis techniques

Tool support
A Petri net system \((P, T, F, m_0)\) consists of a Petri net \((P, T, F)\) and a distinguished marking \(m_0\), the initial marking.

\[
P = \{r1,r2,g1,g2,o1,o2,x,y\} \\
T = \{rg1,rg2,go1,go2,or1,or2\} \\
F = \{(r1,rg1),(y,rg1),(rg1,g1), (g1,go1),(go1,o1), \ldots\} \\
m_0(x) = 1 \\
m_0(r1) = 1 \\
m_0(g1) = 0 \\
\ldots
\]
Transition firing (2)

For a Petri net \((P, T, F)\), let \(w\) be the weight function and let \(m: P \to \mathbb{N}\) be the current marking.

A transition \(t \in T\) can fire if it is enabled at \(m\).

The firing of \(t\) yields a new marking \(m': P \to \mathbb{N}\) where for all places \(p \in P\),

\[
m'(p) = m(p) - w((p, t)) + w((t, p)).
\]

\(t\)
\(p_1\)
\(p_2\)
\(p_3\)

\[
\begin{align*}
m'(p_1) &= 1 - 1 + 0 = 0 \\
m'(p_2) &= 2 - 1 + 0 = 1 \\
m'(p_3) &= 0 - 0 + 1 = 1
\end{align*}
\]
Mapping Petri nets onto transition systems
A Petri net system \((P, T, F, m_0)\) defines the following transition system \((S, TR, s_0)\):

**Intuition**

\[
s_0 = m_0
\]

\[
s = m: P \rightarrow N
\]
A Petri net system \((P, T, F, m_0)\) defines the following transition system \((S, TR, s_0)\):

- \(S = M = P \rightarrow N\)
- \(TR = \{ (m, m') \in S \times S \mid \exists t \in T : (\forall p \in \bullet t : m(p) > 0) \land (\forall p \in P : m'(p) = m(p) - w((p, t)) + w((t, p)) ) \}\)
- \(s_0 = m_0\)
Reachability graph

Recall mapping of a Petri net onto a transition system

$s = m: P \rightarrow N$

Reachability graph is the reachable portion of the transition system (reachable from initial marking)
Reachability graph algorithm

1) Label the initial marking \( m_0 \) as the \textit{root} and tag it "new".

2) While "new" markings exists, do the following:
   a) Select a new marking \( m \).
   b) If no transitions are enabled at \( m \), tag \( m \) "dead-end".
   c) While there exist enabled transitions at \( m \), do the following for each enabled transition \( t \) at \( m \):
      i. Obtain the marking \( m' \) that results from firing \( t \) at \( m \).
      ii. If \( m' \) does not appear in the graph add \( m' \) and tag it "new".
      iii. Draw an arc with label \( t \) from \( m \) to \( m' \) (if not already present).

3) Output the graph
Step 1: Label the initial marking $m_0$ as the root and tag it "new" (indicated by green color).
Example (continued)
Example (continued)
Example (continued)
Example (continued)
Example (continued)
Example (continued)
Example (continued)
Each state is represented as a multiset.

Alternatively, as a vector: 
\((1,0,0,1,1,0,0)\) assuming the ordering \((r1,g1,o1,x,r2,g2,o2)\).
Example

```
Example

[2·wait, free] [wait, busy] [wait, free, done] [free, 2·done]
```

![Diagram of states with transitions](image)

- Start
- Stop
- Busy
- Free
- Done

Transitions:
- Start → [2·wait, free]
- Start → [wait, busy]
- Stop → [wait, free, done]
- Stop → [busy, done]
- Stop → [free, 2·done]
Reachability, run

Marking $m$ reachable if there exists a path from $m_0$ to $m$. 

Path from $m_0$ to $m$ is a run (can be finite or infinite).

Terminal state (deadlock)
Reachability graph and analysis

- Inspecting the reachability graph of a Petri net is one kind of analysis.
- Petri net can have a huge number of reachable markings (state explosion).
- Even an infinite number, see unbounded place p2.
- We will investigate this later in this course.
Using CPN Tools
first steps ...
Modeling Petri nets with CPN Tools
How to get started with CPN Tools?

Videos
Examples

- Getting Started
- Graphical User Interface
  - Graphical User Interface
  - Palette tools
  - Marking menus
- Tasks in CPN Tools
  - Editing a CPN
Example

- A small bank has 3 employees to serve customers.
- An employee can only serve 1 customer at the same time and only one service can be provided (e.g., deposit money).
- Due to space limitations, only 20 customers can be in the bank at the same time.
- After being served a customer can decide to queue again for another service (e.g., after depositing money, the customer wants to change euros into dollars).
- Model as a classical Petri net using CPN Tools.
- How many states does the corresponding transition system have?
Solution

\[
\frac{22!}{20!(2!)} + \frac{21!}{19!(2!)} + \frac{20!}{18!(2!)} + \frac{19!}{17!(2!)} = 802
\]
Example Revisited

• Model a circular railway system with four stations (st1, st2, st3, and st4) and one train.
• At each station passengers may "hop on" or "hop off". This is impossible if the train is moving.
• The train has a capacity of 50 persons; if the train is full, no new passengers may hop on.
• Model the above process in terms of a Petri net.
• What is the number of reachable states?
• Hints:
  − How to describe the state of the train in terms of it location (e.g., moving from st1 to st2) and number of passengers (e.g., 36)?
  − What are possible actions?
  − When are they possible?
Earlier solution

51 x 4 = 204 states
in CPN Tools
Alternative solution
Remember ...

- Most industrial tools have a token-based semantics, for example

- Even if notations look different (BPMN, EPCs, UML ADs, etc.), the basic mechanisms are similar to the first process modeling notation taking concurrency as a starting point (i.e., Petri nets).
## Exercise

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<th>Type</th>
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<th>to prepare</th>
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<td>17</td>
<td>22-4-2013</td>
<td>Lect.</td>
<td>Introduction, transition systems, Petri nets (1)</td>
<td>Read Chapters 1-3 of book. Make all exercises in Section 1 and part of the exercises in Section 2.</td>
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<td>Petri nets (2)</td>
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<td>18</td>
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<td>TU/e closed</td>
<td>Read Chapter 4 of book. Make all exercises in Section 2.</td>
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<td>Lect.</td>
<td>Extending Petri nets with channels</td>
<td>Read Chapter 5 of book.</td>
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After this lecture you should be able to

• Understand how to map a Petri net onto a transition system.
• Construct the corresponding reachability graph for a given Petri net.
• Model a classical Petri net using CPN Tools.